INTERACTION OF CHARGED 3D SOLITON WITH COULOMB CENTER

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The Einstein - de Broglie particle-soliton concept is applied to simulate stationary states of an electron in a hydrogen atom. According to this concept, the electron is described by the localized regular solutions to some nonlinear equations. In the framework of Synge model for interacting scalar and electromagnetic fields a system of integral equations has been obtained, which describes the interaction between charged 3D soliton and Coulomb center. The asymptotic expressions for physical fields, describing soliton moving around the fixed Coulomb center, have been obtained with the help of integral equations. It is shown that the electron-soliton center travels along some stationary orbit around the Coulomb center. The electromagnetic radiation is absent as the Poynting vector has non-wave asymptote $O(r^{-3})$ after averaging over angles, i.e. the existence of spherical surface corresponding to null Poynting vector stream, has been proved. Vector lines for Poynting vector are constructed in asymptotical area.

Key words: soliton, nonlinear resonance, wave-particle dualism, theory of double solution, electromagnetic radiation, stationary orbit

1 Introduction

From the history of quantum mechanics it is known that as early as 1927 in the framework of his "theory of double solution" Louis de Broglie made an attempt to represent the electron as a source of waves obeying the Schrödinger equation [1]. Later he modified his model showing that the electron should be described by regular solutions to some nonlinear equation coinciding with the Schrödinger one in the linear approximation. This scheme became famous as a causal nonlinear interpretation of quantum mechanics [2]. Developing this concept, de Broglie remarked that it had much in common with Einstein's ideas about unified field theory according to which particles were to be considered as clots of some material fields obeying the nonlinear field equations [3]. In recent years, these types of field configurations, known as soliton or particle-like solutions, came into active use to model extended elementary particles [4].

In this paper the Einstein-de Broglie soliton concept is employed to model stationary states of the electron in a hydrogen atom.

2 Bohm problem about nonlinear resonance and its possible solution

As a starting point, we will consider an interesting problem posed by D. Bohm. Long ago, in his book [5] D. Bohm discussed the possible connection between the wave-particle dualism in quantum mechanics and the hypothetical nonlinear origin of fundamental equations in a future theory of elementary particles. To illustrate the line of Bohm's argument we will consider a simple scalar model in the Minkowski space-time given by the Lagrangian density

$$\mathcal{L} = \partial_i\phi^*\,\partial_j\phi\,\eta^{ij} - (mc/\hbar)^2\phi^*\phi + F(\phi^*\phi). \tag{2.1}$$

Here $i, j = 0, 1, 2, 3;\quad \eta^{ij} = \mathrm{diag}(1, -1, -1, -1)$, the nonlinear function $F(s)$ behaves at $s \to 0$ as $s^n, \quad n > 1$, and is assumed as such that the corresponding field equations allow the existence of particle-like (soliton) solutions, i.e. regular configurations localized in space and endowed with finite energy. In particular, it can be shown that if one chooses $F(s) = ks^{3/2}, \quad k > 0$, the model (2.1), known as the Synge one [6], admits the following stationary solutions:

$$\phi_0 = u(r)\exp(-i\omega_0 t), \quad r = |\mathbf{r}|. \tag{2.2}$$

Here, the real radial function $u(r)$ is regular everywhere and exponentially decreases as $r \to \infty$, that provides finiteness of energy of the configuration

$$E = \int d^3x\, T^{00}(\phi_0), \tag{2.3}$$

where T^{ij} is the corresponding energy-momentum tensor.

Moreover the model mentioned is intriguing due to the fact that nodeless solitons turn out to be stable by Lyapunov provided that their charge is fixed [7]. So there exist perturbed solitons slightly differing from the stationary solitons (2.2):

$$\phi = \phi_0 + \xi(t, \mathbf{r}). \tag{2.4}$$

Note that the perturbation ξ in (2.4) is small as compared with ϕ_0 only in the area of localization of the soliton, where ϕ_0 significantly differs from zero. None the less far from

the soliton center, where ϕ_0 is negligibly small, one can put $\phi \approx \xi$, i.e. the *tail* of the soliton is completely defined by the perturbation ξ.

D. Bohm put the following question: Does there exist any nonlinear model for which the spatial asymptote (as $r \to \infty$) of a perturbed soliton-like solution represents oscillations with characteristic frequency $\omega = E/\hbar$? In other words, for the model in question the principal Fourier-amplitude in the expansion of the field $\phi \approx \xi$ as $r \to \infty$ should correspond to the frequency ω connected with the soliton energy (2.3) by the Planck-de Broglie formula

$$E = \hbar\omega. \tag{2.5}$$

Note that for the model (2.1) at spatial infinity, where $\phi \to 0$, the field equation reduces to the linear Klein-Gordon one

$$[\Box - (mc/\hbar)^2]\phi = 0, \tag{2.6}$$

and therefore the relation (2.5) holds only for solitons with unique energy $E = mc^2$ defined by the mass m fixed in (2.1). Thus the universality of the relation (2.5) breaks down in the model (2.1), so forcing its modification. In the light of the above universality, the frequency ω in (2.5) being defined by the mass of the system, it seems natural that in the new, modified model one should use the gravitational field, spatial asymptote of which is also defined by the mass of the considered localized system. Thus, to solve the Bohm problem the possibility to invoke the gravitational field comes into reality [8].

So we will describe the new model with the Lagrangian density $\mathcal{L} = \mathcal{L}_m + \mathcal{L}_g$, where

$$\mathcal{L}_g = c^4 R/16\pi G$$

corresponds to the Einstein's theory of gravity, and $\mathcal{L}_m$ is chosen as

$$\mathcal{L}_m = \partial_i\phi^*\, \partial_j\phi\, g^{ij} - I(g_{ij})\phi^*\phi + F(\phi^*\phi). \tag{2.7}$$

The crucial point of this scheme is to build up the invariant $I(g_{ij})$ depending on the metric tensor g_{ij} of the Riemannian space-time and its derivatives. This invariant should possess such properties that in the vicinity of the soliton with mass m, the relation

$$\lim_{r\to\infty} I(g_{ij}) = (mc/\hbar)^2 \tag{2.8}$$

should hold. It is easy to see that on the basis of (2.8) one can asymptotically deduce the equation (2.6) from the Lagrangian (2.7).

We argue that the invariant I can be built from the curvature tensor R_{ijkl} and its covariant derivatives $R_{ijkl;n}$:

$$I = (I_1^4/I_2^3)c^6\hbar^{-2}G^{-2}, \tag{2.9}$$

where G is the Newton's gravitational constant and the invariants I_1 and I_2 take the form

$$I_1 = R_{ijkl}R^{ijkl}/48, \quad I_2 = -R_{ijkl;n}R^{ijkl;n}/432.$$

Estimating R^{ijkl} at large distance r with the help of the Schwarzschild metric, one can find

$$I_1 = G^2m^2/(c^4r^6), \quad I_2 = G^2m^2/(c^4r^8).$$

So from (2.9) there immediately follows (2.8). Thus, within the modified model (2.7) for all massive particles the Planck-de Broglie relation (2.5) is automatically fulfilled. It means that in the framework of the scheme mentioned the principle of wave-particle

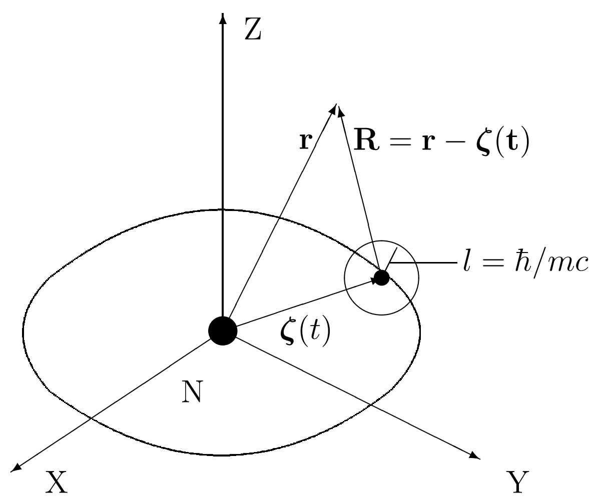
dualism is valid, according to which the relation (2.5) is realized as a condition of the nonlinear resonance.

To verify the fact that solitons can really possess wave properties, the *gedanken* diffraction experiment with individual electron-solitons similar to the numerical one of Biberman et al. [9], was realized. Solitons with some velocity were dropped into a rectilinear slit, cut in the impermeable screen, and the transverse momentum was calculated which they gained while passing the slit the width of which significantly exceeded the size of the soliton. As a result, the picture of distribution of the centers of scattered solitons was restored on the registration screen, by considering their initial distribution to be uniform over the transverse coordinate. It was clarified that though the center of each soliton fell into a definite place of the registration screen (depending on the point of crossing of the slit and the initial soliton profile), the statistical picture in many ways was similar to the well-known diffraction distribution in optics, i.e. Fresnel's picture at short distances from the slit and Fraunhofer's picture at large distances [10,11].

Fulfillment of the quantum mechanics correspondence principle for the Einstein-de Broglie's soliton model was discussed in the works [12-15]. In these papers it was shown that in the framework of the soliton model all quantum postulates were regained at the limit of point particles so that from the physical fields one can build the amplitude of probability and the average can be calculated as a scalar product in the Hilbert space by introducing the corresponding quantum operators. In this paper, we will show that in the framework of the Einstein-de Broglie soliton model a hydrogen atom can be simulated.

3 Fundamental equations and structure of solutions

Let us consider the hydrogen atom with the electron replaced by a localized object "soliton" that is moving round the nucleus. So that the soliton-like solution does exist one has to consider a nonlinear model.



As physical fields we choose the complex scalar field ϕ interacting with the electromagnetic one $F_{ik} = \partial_i A_k - \partial_k A_i$. The nucleus field is assumed to be the Coulomb one: $A_i^{ext} = \delta_i^0 Ze/r$. The Lagrangian density is taken in the following form

$$\mathcal{L} = -\frac{1}{16\pi}(F_{ik})^2 + |[\partial_k - i\epsilon(A_k + A_k^{ext})]\phi|^2 - (mc/\hbar)^2\phi^*\phi + F(\phi^*\phi), \qquad (3.1)$$

where $\epsilon = e/(\hbar c)$ is the coupling constant, $F(\phi^*\phi)$ is some nonlinear function, decreasing faster as $\phi \to 0$ than $|\phi|^2$ and is chosen so that the field equation at $A_i^{ext} = 0$ allows the existence of stable stationary soliton-like solutions of the type (2.2), describing configurations with mass m and charge e.

Note that for simplicity we do not write down the terms corresponding to the gravitational field that will be taken into account implicitly with the help of the nonlinear resonance condition (2.5).

Let us consider the nonrelativistic approximation assuming that

$$\phi = \psi \, \exp{(-imc^2t/\hbar)}, \tag{3.2}$$

neglecting in the equations of motion higher derivatives of ψ with respect to time and retaining only linear terms in A_i. As a result, taking (3.2) into account we get the following system of equations:

$$i\hbar \, \partial_t\psi + (\hbar^2/2m)\triangle\psi + (Ze^2/r)\psi \; = \; -(\hbar^2/2m)\hat{f}(\mathbf{A}, A_0, \psi^*\psi)\psi, \tag{3.3}$$

$$\Box A_0 = (8\pi me/\hbar^2)|\psi|^2 \equiv -4\pi\rho, \tag{3.4}$$

$$\Box\mathbf{A} = 4\pi[2\epsilon^2\mathbf{A}|\psi|^2 - i\epsilon(\psi^*\nabla\psi - \psi\nabla\psi^*)] \equiv -(4\pi/c)\,\mathbf{j}, \tag{3.5}$$

where

$$\hat{f}(\mathbf{A}, A_0, \psi^*\psi)\psi \; \equiv \; 2i\epsilon(\mathbf{A}\nabla)\psi + 2(\epsilon mc/\hbar)A_0\psi + i\epsilon\psi \, \mathrm{div}\mathbf{A} + F'(\psi^*\psi)\psi.$$

Moreover in the equations (3.3 - 3.5) it is supposed that the 4-potential A_i of the proper electromagnetic field of the soliton obeys to the Lorentz condition

$$\partial_t A_0 + \mathrm{c} \; \mathrm{div}\mathbf{A} = 0,$$

which is consistent with equations (3.3) - (3.5) owing to the conservation of electric charge.

We will seek for the solutions to equations (3.3) - (3.5) describing the stationary state of an atom when the electron-soliton center is assumed to be moving along a circular orbit of the radius a_0 with some angular velocity Ω. In this problem there arise two characteristic lengths: the size of the soliton $l = \hbar/(mc)$ and the Bohr's radius $a = \hbar^2/(mZe^2)$. It is obvious that $a_0 \sim a \gg l$.

Let us first consider the area near the soliton center where $r - a_0 \sim l$. Suppose the soliton center trajectory to be $\mathbf{r} = \boldsymbol{\zeta}(t)$. Putting into (3.3) the configuration

$$\psi = u(\mathbf{r} - \boldsymbol{\zeta}(t)) \, \exp{(i\mathcal{S}/\hbar)},$$

neglecting the contribution of the proper electromagnetic field and separating the real and imaginary parts, we get

$$\partial_t\mathcal{S} - \frac{Ze^2}{r} + \frac{1}{2m}(\nabla\mathcal{S})^2 - \frac{\hbar^2}{2m}(\hat{f} + \frac{\triangle u}{u}) = 0, \tag{3.6}$$

$$\triangle\mathcal{S} + 2(\nabla\mathcal{S} - m\dot{\boldsymbol{\zeta}}) \cdot \nabla u/u = 0. \tag{3.7}$$

Assuming S to be a slowly varying function of a point in the vicinity of the soliton center, from (3.7) we deduce

$$\mathcal{S} \approx m\dot{\boldsymbol{\zeta}} \cdot (\mathbf{r} - \boldsymbol{\zeta}) + C_0 t + \chi(t), \quad C_0 = \mathrm{const}. \tag{3.8}$$

Taking into account the classical equations of motion of a charged particle in the Coulomb field

$$m\ddot{\boldsymbol{\zeta}} = -Ze^2\boldsymbol{\zeta}/\zeta^3$$

and using the expansion

$$\frac{1}{r} \approx \frac{1}{\zeta} - \frac{\boldsymbol{\zeta}\cdot(\mathbf{r}-\boldsymbol{\zeta})}{\zeta^3},$$

from (3.6) and (3.8) we derive

$$\partial_t\chi = \frac{m}{2}\dot{\boldsymbol{\zeta}}^2 + \frac{Ze^2}{\zeta} \equiv \mathcal{L}(t),$$

where $\mathcal{L}(t)$ is the Lagrangian of a particle in the Coulomb field. Thus, the function χ is the classical action on the trajectory:

$$\chi(t) = \int\limits_0^t \mathcal{L}(t)dt, \tag{3.9}$$

and the function u is the soliton-like solution to the quasi-stationary problem

$$\hbar^2(\hat{f} + \triangle u/u) = 2mC_0. \tag{3.10}$$

In this case according to (3.4) and (3.5)

$$\rho = -(2me/\hbar^2)u^2, \quad \mathbf{j} = -2\epsilon cu^2(\epsilon\mathbf{A} + m\dot{\boldsymbol{\zeta}}/\hbar),$$

which makes it possible, using the common solutions to equations (3.4), (3.5) and (3.10), to calculate the potentials A_i of the electromagnetic field in the vicinity of the soliton center:

$$A_0 = A_0(\mathbf{r}-\boldsymbol{\zeta}(t)), \quad c\mathbf{A} = \dot{\boldsymbol{\zeta}}(t)A_0(\mathbf{r}-\boldsymbol{\zeta}(t)),$$

where the terms $\dot{\boldsymbol{\zeta}}^2/c^2$ are neglected.

Let us now study the asymptotic behavior of soliton at large distance, i.e. we will study it's *"tail"*. We will use successive approximation method. Thus, we need to rewrite the differential equations (3.3), (3.4) and (3.5) in integral form. Note that we are not solving Cauchy evolution problem, choosing definite initial condition. Were it a question of Cauchy problem, we would bound to use retarded Green's function. In this case it has been assumed that the object under consideration (atom) already exists infinitely long. Thus we consider the problem to study corresponding steady state, which eliminates the possibility of using retarded electromagnetic Green's function. The problem mentioned, according to our view, can be satisfied by half-sum of retarded and advanced solutions. Above mentioned selection can be justified by the assumption that during the evolution process the radiation of the independent i.e. half-difference of retarded and advanced electromagnetic fields took place.

To find out the field ψ far from the soliton center, we rewrite equation (3.3) in the integral form

$$\begin{aligned}\psi(t,\mathbf{r}) &= C_n\psi_n(\mathbf{r})\exp(-i\omega_n t) + \\ &+ \frac{1}{2\pi}\int d\omega\int dt'\int d^3x' \exp[-i\omega(t-t')]\, G(\mathbf{r},\mathbf{r}';\omega+i0)\hat{f}\psi(t',\mathbf{r}'),\end{aligned} \tag{3.11}$$

where $\psi_n(\mathbf{r})$ is the eigenfunction of the Hamiltonian of a hydrogen atom for a stationary state of number n with energy $E_n = \hbar\omega_n, \quad C_n = \text{const}$ and $G(\mathbf{r}, \mathbf{r}'; \omega)$ is the Hamiltonian's resolvent having the form [16]

$$G(\mathbf{r}, \mathbf{r}'; \omega) = \frac{\Gamma(1-i\nu)}{4\pi R} \begin{vmatrix} W_{i\nu,1/2}(-ikr_+) & M_{i\nu,1/2}(-ikr_-) \\ \dot{W}_{i\nu,1/2}(-ikr_+) & \dot{M}_{i\nu,1/2}(-ikr_-) \end{vmatrix}. \tag{3.12}$$

Here,the following notation is used:

$$k = (2m\omega/\hbar)^{1/2}, \quad Im\, k > 0, \quad \nu = (ka)^{-1},$$

$$r_\pm = r + r' \pm |\mathbf{r} - \mathbf{r}'|$$

and the Whittaker functions $W_{i\nu,1/2}, M_{i\nu,1/2}$ and their derivatives $\dot{W}_{i\nu,1/2}$, $\dot{M}_{i\nu,1/2}$ are introduced. To find the field ψ at large distances from the electron-soliton center, i.e. at $|r - a_0| \gg l$, it is sufficient to put in (3.11)

$$\hat{f}\,\psi(t, \mathbf{r}) = g\, \exp\left(-i\omega_n t\right) \delta(\mathbf{r} - \boldsymbol{\zeta}(t)), \quad g = const, \tag{3.13}$$

where the relation (2.5) is taken into account. As a result, we get

$$\begin{aligned} \psi(t, \mathbf{r}) &= C_n \psi_n(\mathbf{r}) \exp\left(-i\omega_n t\right) + \\ &+ \frac{1}{2\pi} \int d\omega \int dt' \exp\left[-i\omega\, t + it'(\omega - \omega_n)\right] G(\mathbf{r}, \mathbf{r}'; \omega + i0). \end{aligned} \tag{3.14}$$

It is easy to verify that the field (3.14) decreases exponentially at large distances. With the help of (3.14) and equations (3.4) and (3.5), one can evaluate the electromagnetic field outside the soliton. In (3.14) C_n is unknown constant and $\psi_n(\mathbf{r})$ is the wave function of electron in stationary state. In each step of iteration one obtains (3.14) where the stationary "tail" of soliton is marked out and its center moves along some effective orbit. The orbital parameter and constant C_n may be defined in arbitrary approximation of minimization $||\psi_{(k)} - \psi_{(k+1)}||$ [17]. Still now the constants C_n and g are not found explicitly and we hope to obtain them in our forthcoming papers.

Let us now solve the equations (3.4) and (3.5). Considering that the nonlinear source is rather weak one, we will replace the right hand sides of the equations (3.4) and (3.5) by δ- functions. Let us also notice that

$$\mathbf{E}_- \;=\; (\mathbf{E}_- + \mathbf{E}_+)/2 \;+\; (\mathbf{E}_- - \mathbf{E}_+)/2,$$

where $\mathbf{E}_- = \mathbf{E}^{ret}, \quad \mathbf{E}_+ = \mathbf{E}^{adv}$. It is well known that the half-difference of retarded and advanced fields radiates. So it will be sufficient to consider, as was discussed earlier, the half-sum of retarded and advanced solutions, describing the stationary state. It means, we will seek the strength of the electromagnetic field as the half-sum of those for retarded and advanced fields. It means that for large times $|\omega_n|t \gg 1$ the 4-potential A_k will contain only stationary part $A_k = (A_k^{ret} + A_k^{adv})/2$.

Let us find the expression for $\mathbf{E}_-$. Radius of the soliton l is rather small in comparison to Bohr radius a, i.e. $a \gg l$. So the source can be considered as proper one. Let the point-like charge e move along the given trajectory $\mathbf{r} = \boldsymbol{\zeta}(\mathbf{t})$ with velocity $\mathbf{v}(\mathbf{t}) = \dot{\boldsymbol{\zeta}}(\mathbf{t})$. Then, to describe the electromagnetic field, generated by the charge, one can write charge density and current density as

$$\rho(t, \mathbf{r}) = e\, \delta[\mathbf{r} - \boldsymbol{\zeta}(t)], \quad \mathbf{j}(t, \mathbf{r}) = e\mathbf{v}(t)\, \delta[\mathbf{r} - \boldsymbol{\zeta}(t)]. \tag{3.15}$$

Then the equations (3.4) and (3.5) take form:

$$\begin{aligned}\Box A_0 &= -4\pi e\,\delta[\mathbf{r}-\boldsymbol{\zeta}(t)],\\ \Box \mathbf{A} &= -\frac{4\pi e\mathbf{v}}{c}\,\delta[\mathbf{r}-\boldsymbol{\zeta}(t)].\end{aligned}$$

that lead to the well-known Lienard- Wiechert potentials. As we know, the retarded time writes

$$t_- = t - R(t_-,\mathbf{r})/c,$$

which leads to

$$\frac{\partial t_-}{\partial t} = \frac{1}{1-(\mathbf{n}_-\cdot\mathbf{v})/c}, \quad \frac{\partial \mathbf{R}(t_-,\mathbf{r})}{\partial t_-} = -(\mathbf{n}_-\cdot\mathbf{v}), \quad \nabla t_- = -\frac{\mathbf{n}_-}{c-(\mathbf{n}_-\cdot\mathbf{v})}.$$

where $\mathbf{n} = \mathbf{R}(t,\,\mathbf{r})/R(t,\,\mathbf{r})$. Later, using the following expressions for $A_{0-}(t,\mathbf{r})$ and $\mathbf{A}_-(t,\mathbf{r})$ [18, 19]

$$A_{0-}(t,\mathbf{r}) = \frac{ec}{[cR-(\mathbf{v}\cdot\mathbf{R})]_{t_-}}, \quad \mathbf{A}_-(t,\mathbf{r})\left[\frac{e\mathbf{v}}{cR-(\mathbf{v}\cdot\mathbf{R})}\right]_{t_-},$$

one finds the expression for retarded field

$$\mathbf{E}_- = e\left[\frac{(c^2-v^2)(c\mathbf{n}_- -\mathbf{v})}{R^2[c-(\mathbf{n}_-\cdot\mathbf{v})]^3} + \frac{[\mathbf{n}_-,[(c\mathbf{n}_- -\mathbf{v}),\dot{\mathbf{v}}]]}{R[c-(\mathbf{n}_-\cdot\mathbf{v})]^3}\right]_{t_-}, \tag{3.16}$$

and

$$\mathbf{B}_- = [\mathbf{n}_-,\mathbf{E}_-]_{t_-}. \tag{3.17}$$

Analogically writing the advanced time as

$$t_+ = t + R(t_+,\mathbf{r})/c,$$

for the advanced field we find

$$\mathbf{E}_+ = e\left[\frac{(c^2-v^2)(c\mathbf{n}_- +\mathbf{v})}{R^2[c+(\mathbf{n}_-\cdot\mathbf{v})]^3} + \frac{[\mathbf{n}_-,[(c\mathbf{n}_- +\mathbf{v}),\dot{\mathbf{v}}]]}{R[c+(\mathbf{n}_-\cdot\mathbf{v})]^3}\right]_{t_-}, \tag{3.18}$$

and

$$\mathbf{B}_- = -[\mathbf{n}_+,\mathbf{E}_+]_{t_+}. \tag{3.19}$$

To calculate the power, lost by the charge due to radiation, one has to compose Poynting vector $\mathbf{S}$ and retain the terms of the order $1/R^2$ as the integration will take place along infinitely distant surface. As we know, Poynting vector is expressed by the relation

$$\mathbf{S} = \frac{c}{4\pi}[\mathbf{E},\mathbf{B}]. \tag{3.20}$$

In this case the field strengths are

$$\mathbf{E} = \frac{1}{2}(\mathbf{E}_+ + \mathbf{E}_-), \quad \mathbf{B} = \frac{1}{2}([\mathbf{n}_-,\mathbf{E}_-] - [\mathbf{n}_+,\mathbf{E}_+]), \tag{3.21}$$

where $\mathbf{E}_- = \mathbf{E}^{ret}$, $\mathbf{E}_+ = \mathbf{E}^{adv}$, $\mathbf{n}_\pm = \mathbf{R}_\pm/R_\pm$, $\mathbf{R}_\pm = \mathbf{r} - \boldsymbol{\zeta}(t_\pm)$ and $t_\pm$ are the roots of the equations $t_\pm = t \pm R_\pm/c$.

Using some manipulations from vector analysis we rewrite Poynting vector as

$$\mathbf{S} = \frac{c}{16\pi}\{\mathbf{n}_-\mathbf{E}_-^2 - \mathbf{n}_+\mathbf{E}_+^2 + (\mathbf{n}_- - \mathbf{n}_+)(\mathbf{E}_- \cdot \mathbf{E}_+) - \\ -\mathbf{E}_-((\mathbf{E}_- + \mathbf{E}_+)\cdot \mathbf{n}_-) + \mathbf{E}_+((\mathbf{E}_- + \mathbf{E}_+)\cdot \mathbf{n}_+)\}. \tag{3.22}$$

Let us now rewrite all these in spherical system of coordinates. In this case

$$\begin{aligned}
\mathbf{R}(\mathbf{r}, t_\pm) &= \{r - a_0 \sin\theta \cos\alpha_\pm,\ -a_0 \cos\theta \cos\alpha_\pm,\ a_0 \sin\alpha_\pm\}, \\
\mathbf{v}(t_\pm) &= \{a_0\,\Omega \sin\theta \sin\alpha_\pm,\ a_0\,\Omega \cos\theta \sin\alpha_\pm,\ a_0\,\Omega \cos\alpha_\pm\}, \\
\dot{\mathbf{v}}(t_\pm) &= \{-a_0\,\Omega^2 \sin\theta \cos\alpha_\pm,\ -a_0\,\Omega^2 \cos\theta \cos\alpha_\pm,\ a_0\,\Omega^2 \sin\alpha_\pm\}, \\
R(\mathbf{r}, t_\pm) &= \sqrt{r^2 + a_0^2 - 2a_0 r \sin\theta \cos\alpha_\pm},
\end{aligned}$$

where $\alpha_\pm = \alpha - \Omega\, t_\pm$. Retaining the terms of the order a_0/r for $\mathbf{n}_-$ and $\mathbf{n}_+$ one can obtain

$$\mathbf{n}_\pm = \{1, -(a_0/r)\cos\theta \cos(\alpha - \Omega t_\pm), (a_0/r)\sin(\alpha - \Omega t_\pm)\}.$$

Using the first nonvanishing approximation of the order v/c, from (3.16) and (3.18) we will get

$$\mathbf{E}_- \approx e\Big[\frac{\mathbf{n}_-}{R^2} + \frac{\mathbf{n}_-(\mathbf{n}_- \cdot \dot{\mathbf{v}}) - \dot{\mathbf{v}}}{c^2 R}\Big]_{t_-}, \tag{3.23}$$

$$\mathbf{E}_+ \approx e\Big[\frac{\mathbf{n}_+}{R^2} + \frac{\mathbf{n}_+(\mathbf{n}_+ \cdot \dot{\mathbf{v}}) - \dot{\mathbf{v}}}{c^2 R}\Big]_{t_+}. \tag{3.24}$$

Taking into account that

$$1/R \approx 1/r - (a_0/r^2)\sin\theta \cos\alpha_-, \quad 1/R^2 \approx 1/r^2, \\ (\mathbf{n}_- \cdot \dot{\mathbf{v}}) \approx -a_0\Omega^2 \sin\theta \cos\alpha_- + (a_0^2\Omega^2/r)[1 - \sin^2\theta \cos^2\alpha_-],$$

where $\alpha_- = \alpha - \Omega t_-$, for $\mathbf{E}_-$ one gets

$$\mathbf{E}_- \approx e\{\frac{1}{r^2}[1 + (a_0^2\Omega^2/c^2)(1 - \sin^2\theta \cos^2\alpha_-)], (a_0\Omega^2/c^2 r)\cos\theta \cos\alpha_-,\ -(a_0\Omega^2/c^2 r)\sin\alpha_-\}.$$

Analogically one finds

$$\mathbf{E}_+ \approx e\{\frac{1}{r^2}[1 + (a_0^2\Omega^2/c^2)(1 - \sin^2\theta \cos^2\alpha_+)], (a_0\Omega^2/c^2 r)\cos\theta \cos\alpha_+,\ -(a_0\Omega^2/c^2 r)\sin\alpha_+\}.$$

Putting the above expressions for $\mathbf{n}_\pm, \mathbf{E}_\pm$ into (3.22) one can find the expressions for Poynting vector. In doing so we will take into account that the normals $\mathbf{n}_-$ and $\mathbf{n}_+$ coincide as $r \to \infty$ with $\mathbf{n} = \mathbf{r}/r$. Retaining the terms $(a_0/r)^3$ and also taking into account that $a_0\Omega = v \ll c$, for the circular motion in the spherical coordinates r, θ and α we have the following components of the Poynting vector $\mathbf{S}$:

$$\begin{aligned}
S_r &= \frac{e^2\, a_0^2\, \Omega^4}{16\pi c^3\, r^2} \sin^2\theta \sin 2(\alpha - \Omega t) \sin(2\Omega r/c), \\
S_\theta &= \frac{e^2\, a_0\, \Omega^2}{4\pi c\, r^3} \cos\theta \sin(\alpha - \Omega t) \sin(\Omega r/c), \\
S_\alpha &= \frac{e^2\, a_0\, \Omega^2}{4\pi c\, r^3} \cos(\alpha - \Omega t) \sin(\Omega r/c).
\end{aligned} \tag{3.25}$$

It is obvious that for $r_k = ck\pi/\Omega$ with $k = 0, 1, 2, \cdots$ all the components of Poynting vector turn to zero i.e. $\mathbf{S} = 0$.

All the calculations done above can be summed up as follows. From (3.21) it follows that the projection of the Poynting vector $\mathbf{S}$ in the direction of the vector $\mathbf{N} = (\mathbf{n}_+ + \mathbf{n}_-)/2$, coinciding as $r \to \infty$ with $\mathbf{n} = \mathbf{r}/r$, takes the form

$$S_N = \frac{c}{16\pi\sqrt{2}}(E_-^2 - E_+^2)(1 + \mathbf{n}_+ \cdot \mathbf{n}_-)^{1/2}. \qquad (3.26)$$

Since $\mathbf{n}_\pm = \mathbf{n} + O(r^{-1})$, after averaging expression (3.26) over the sphere, we find

$$< S_N >= \frac{c}{16\pi}\Big(< E_-^2 > - < E_+^2 >\Big) = O(r^{-3}). \qquad (3.27)$$

Thus according to (3.27) the electromagnetic radiation from the system is absent. In particular, for the circular motion in the spherical coordinates r, θ and α we have the following structure of the Poynting vector $\mathbf{S}$:

$$\begin{aligned} S_r &= \frac{\kappa}{r^2}\sin^2\theta \sin 2(\alpha - \Omega t)\sin(2\Omega r/c), \qquad (3.28) \\ S_\theta &= \sin(\Omega r/c)\, O(r^{-3}), \quad S_\alpha = \sin(\Omega r/c)\, O(r^{-3}), \end{aligned}$$

where $\kappa = e^2 a_0^2 \Omega^4/(16\pi c^3)$. From (3.28) as well as from (3.25) it is obvious that there exist spherical surfaces where either $S_r = 0$ or $\mathbf{S} = 0$, thus once again confirming the fact that in the stationary states described, radiation is absent [20].

Let us describe the vector lines for Poynting vector. In spherical system of coordinates we have

$$\frac{dr}{S_r} = r\,\frac{d\theta}{S_\theta} = r\sin\theta\,\frac{d\alpha}{S_\alpha}. \qquad (3.29)$$

Last two fractions form integrable combination. Putting S_θ and S_α, from this equality we obtain

$$\frac{d\theta}{\sin\theta\cos\theta} = \tan\alpha\, d\alpha, \qquad (3.30)$$

which leads to the first integral

$$|\tan\theta\cos(\alpha - \Omega t)| = P_1, \quad P_1 = \text{const}. \qquad (3.31)$$

Now we consider first two fractions. Putting $\cos(\alpha - \Omega t) = P_1/\tan\theta$ we find

$$\frac{c^2}{\Omega^2}\,\frac{dr}{r^2\cos(\Omega r/c)} = P_1 \sin\theta\, d\theta. \qquad (3.32)$$

As we consider the region where $r \to \infty$ it is possible to factor out the term $1/r^2$ from the integrand. Then we obtain appoximately

$$\frac{c^2}{\Omega^2 r^2}\int \frac{dr}{\cos(\Omega r/c)} = P_1 \int \sin\theta\, d\theta, \qquad (3.33)$$

that leads to

$$\frac{c^2}{\Omega^2 r^2}\ln|\tan(\pi/4 + \Omega r/2c)| = -P_1\cos\theta + P_2, \quad P_2 = \text{const}. \qquad (3.34)$$

Thus, we built vector lines for Poynting vector in asymptotic region.

4 Conclusion

In the considered soliton model of a hydrogen atom the stability condition of spatial stationary motions of electrons in the field of the Coulomb center is fulfilled. The existence of this kind of motion had also been anticipated by Boguslavsky [21] and Chetaev [22]. In particular, due to the fulfillment of the nonlinear resonance condition (2.5) the energy spectrum of these stationary states coincides with that of a hydrogen atom. This fact indicates the role of nonlinearity in the formation of extended micro-objects, whose laws of evolution agree with quantum mechanics.

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